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Mechanical Equilibrium

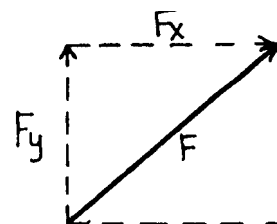
1-1 Forces in Equilibrium

Vocabulary **Force:** A push or a pull.

A force is needed to change an object's state of motion. The downward force acting on an object is called **weight**, while any stretching force on an object (such as a force exerted by a rope or wire) is called **tension**.

The SI (Système International) unit for force is the **newton** (N). A newton weighs about the same as a stick of butter or an apple.

When all the forces acting on an object are balanced and the object is not speeding up or slowing down, the forces are said to be in equilibrium. In order to solve mechanical equilibrium exercises in which the forces are acting at angles to the object, you must study all the forces in the horizontal or x -direction separately from all the forces in the vertical or y -direction. To do this, draw a rectangle with horizontal and vertical sides, where the diagonal is equal to the force you are trying to find. This diagonal is called the resultant, or the sum of the horizontal and vertical components.



Solving this type of exercise sometimes requires an understanding of simple trigonometry. If you are not familiar with trigonometry or need a quick refresher, refer to **Appendix A**. If you are not yet ready to solve problems using trigonometry, return to these problems at a later time.

(Hint: Several of the following exercises involve objects hanging from ropes or wires. Whenever this situation occurs, the sum of the vertical components of the tension in each wire is equal to the object's weight. If the object hangs in the middle of two equal length wires, the weight is shared equally by each wire.)

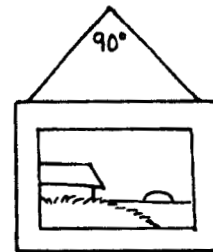
Solved Examples

Example 1: Flip, an exhausted gymnast, hangs from a bar by both arms in an effort to catch his breath. If Flip weighs 650. N, what is the tension in each of Flip's arms as he hangs in place?

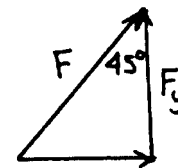
Solution: Since Flip is pulling down on the bar with a force of 650. N, his arms must be holding him up by sharing an upward force of 650. N. If each of Flip's arms shares the force equally, then each must provide a tension of 325 N.

Example 2: At an art auction, Whitney has acquired a painting that now hangs from a nail on her wall, as shown in the figure. If the painting weighs 126 N, what is the tension in each side of the wire supporting the painting?

Solution: The weight of the painting is shared equally by two wires, so each wire must support only half of the weight. However, in this example the wires do not hang vertically, but instead act at an angle with the painting. Therefore, you must use trigonometry to find the actual tension in the wire.



Since the weight of the painting is 126 N, each wire shares 63 N vertically. Call this value F_y , and use trigonometry to find the resulting force.

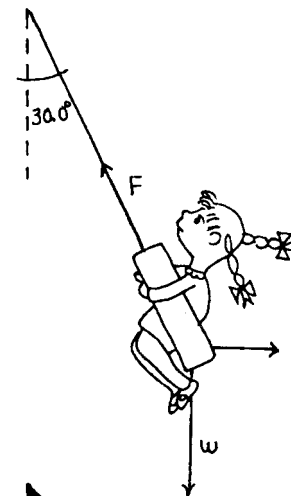


$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{F_y}{F} \quad F = \frac{F_y}{\cos \theta} = \frac{63 \text{ N}}{\cos 45^\circ} = \frac{63 \text{ N}}{0.71} = 89 \text{ N}$$

Therefore, each wire holds the painting with a tension or force of 89 N.

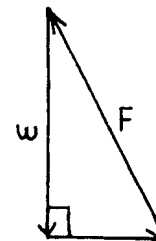
Example 3: Michelle likes to swing on a tire tied to a tree branch in her yard, as in the figure. a) If Michelle and the tire have a combined weight of 825 N, and Michelle's friend Elwin pulls her back far enough for her to make an angle of 30.0° with the vertical, what is the tension in the rope supporting Michelle and the tire?

First, redraw the forces so that they are connected head to tail in a triangle. This allows you to use rules of trigonometry to solve for the tension, F .

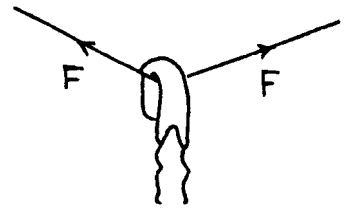


$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{w}{F}$$

$$F = \frac{w}{\cos \theta} = \frac{825 \text{ N}}{\cos 30.0^\circ} = \frac{825 \text{ N}}{0.866} = 953 \text{ N}$$



Example 4: After returning home from the beach, Samantha hangs her wet 2.0-N bathing suit in the center of the 6.0-m-long clothesline to dry. This causes the clothesline to sag 4.0 cm. What is the tension in the clothesline?



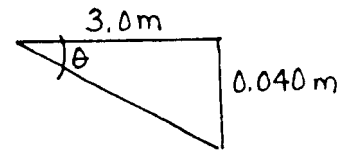
Solution: First, convert cm to m.

$$4.0 \text{ cm} = 0.040 \text{ m}$$

Because the bathing suit is hung in the center of the clothesline, the tension in each side of the line is the same. You must find the downward force on the clothesline, which is simply the weight of the bathing suit, or 2.0 N.

Half of this force, 1.0 N, will pull on the left side of the clothesline and half will pull on the right, *only* because the bathing suit hangs in the middle of the line.

Before you can calculate the tension, F , in the rope, you need to determine the angle the clothesline makes with the horizontal. To do this, use the known distances of 3.0 m and 0.040 m as shown in the diagram. This diagram is not drawn to scale.



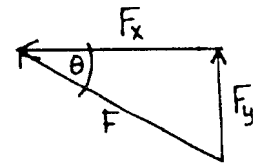
(Not drawn to scale)

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{d_y}{d_x} = \frac{0.040 \text{ m}}{3.0 \text{ m}} = 0.013$$

Taking the inverse of the tangent gives the angle,

$$\tan^{-1} 0.013 = \theta \quad \text{so} \quad \theta = 0.74^\circ$$

The angle is 0.74° and the bathing suit causes the clothesline to pull up with a force of 1.0 N. Now find the tension in the line. Again, this is done using trigonometry, because the angle and the vertical component of the force are known.



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{F_y}{F}$$

$$F = \frac{F_y}{\sin \theta} = \frac{1.0 \text{ N}}{\sin 0.74^\circ} = \frac{1.0 \text{ N}}{0.013} = 77 \text{ N}$$

The tension in each side of the clothesline is 77 N.

Practice Exercises

Exercise 1: While moving out of her dorm room, Bridget carries a 120-N box to her car, holding it in both arms. a) How much force must be exerted by each of her arms to support the box? b) How will this force change if Bridget holds the box with only one arm?

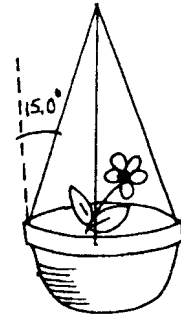
- a) Each arm holds half the weight or **60 N**.
b) The force will be twice as great, or the full **120 N**.

Answer: a. _____ **60 N** _____

Answer: b. _____ **120 N** _____

Exercise 2: A flower pot weighing 42.0 N is hung above a window by three ropes, each making an angle of 15.0° with the vertical, as shown. What is the tension in each rope supporting the flower pot?

- Each rope supports $\frac{1}{3}$ of 42.0 N, or 14.0 N.
 $(14.0 \text{ N})/\cos 15.0^\circ = \mathbf{14.5 \text{ N}}$



Answer: _____ **14.5 N** _____

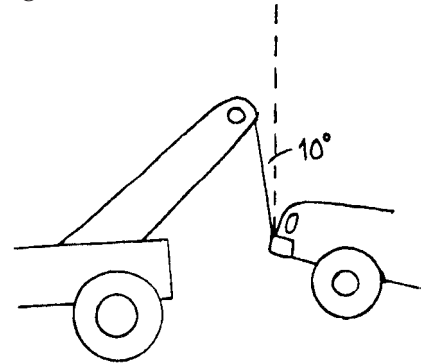
Exercise 3: Luke Skywalker must swing Princess Leia across a large chasm in order to escape the Storm Troopers. If Luke and Leia's combined weight is 1450 N, calculate the tension in the rope just before Luke and Leia start their swing, when the pair makes an angle of 30.0° with the vertical.

$$F = w/\cos \theta = 1450 \text{ N}/\cos 30.0^\circ = \mathbf{1670 \text{ N}}$$

Answer: _____ **1670 N** _____

Exercise 4: The ACE towing company tows a disabled 10,500 N automobile off the road at a constant speed. If the tow line makes an angle of 10.0° with the vertical as shown, what is the tension in the line supporting the car?

$$F = w / \cos \theta = 10,500 \text{ N} / \cos 10.0^\circ = 10,600 \text{ N}$$



Answer: 10,600 N

Exercise 5: Yvette hangs a 24-N bird feeder in the middle of a rope tied between two trees. The feeder creates a tension of 480 N in each side of the rope.
 a) If the two trees are 4.0 m apart, how much will the rope sag in the center?
 b) If a bird lands on the feeder, will this have any effect on the tension in the rope? Explain.

a) $\sin \theta = F_y / F = (12 \text{ N}) / (480 \text{ N}) \quad \theta = 1.43^\circ$

Shared equally by either side of the rope.

$$d_y = (2.0 \text{ m}) \tan 1.43^\circ = 0.050 \text{ m}$$

b) The tension in each side of the rope will be greater to support the bird's weight.

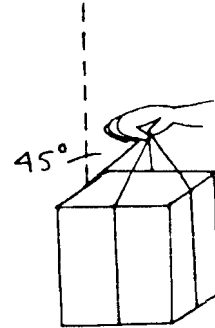
Answer: a. 0.050 m

Answer: b. tension will be greater

Additional Exercises

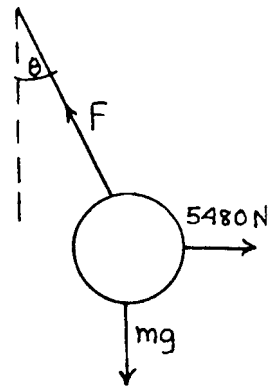
A-1: To make extra money during the summer, Mr. Garber, a 660-N physics teacher, paints the outside of houses while sitting on a 40.-N plank suspended by two vertical cables. What is the tension in each of the two cables?

A-2: Erma receives a 50.0-N package in the mail tied with a string that goes around each side of the box, as shown. If Erma lifts the box by the string in the center so that each piece of string makes an angle of 45.0° with the vertical, what is the tension in each piece of string?



A-3: While camping in Denali National Park in Alaska, a wise camper hangs his pack of food from a rope tied between two trees, to keep the food away from the bears. If the 50.-N bag of food hangs from the center of a rope that is 3.0 m long, and the rope sags 6.0 cm in the middle, what is the tension in the rope?

A-4: In the figure, a 12,400-N wrecking ball is pulled back with a horizontal force of 5480 N before being swung against the side of a building. a) What angle does the wrecking ball make with the vertical when it is pulled back? b) What is the tension in the ball's supporting cable when it is at this angle?

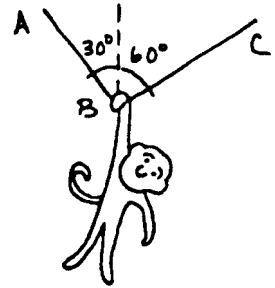


A-5: The amazing Gambini walks across a 30.0-m-long tightrope high above a 3-ring circus. If the 750.-N Gambini pushes the tightrope down 15.0 cm in the center, find the tension in the tightrope.

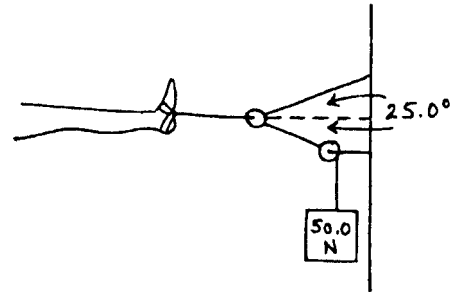


Challenge Exercises for Further Study

- B-1:** Malcolm, the 200.-N monkey, hangs from a jungle vine, as shown. a) What is the tension in the segment of vine labeled AB? b) What is the tension in the segment of the vine labeled BC?



- B-2:** Blythe lies in a hospital bed with her foot in traction, as shown. How much tension will the traction device exert on her foot?



- B-3:** After pulling his car off to the side of the road during a rainstorm, Travis is dismayed to find that the car has become stuck in the mud. Travis ties one end of a rope to the front of the car and the other end to a tree 12.00 m away as shown. a) If Travis can exert a force of 610 N on the rope, moving it 1.00 m in the direction shown, how much force will the rope exert on the car? b) Why is this method better than simply tying a rope to the front of the car and pulling the car straight out?

